SCOTCH COLLEGE

# Practise Exam 1 Semester 2, 2016
# MATHEMATICS: SPECIALIST

**Question/Answer Booklet – Section 2 – Calculator-assumed**

Teacher's Name: ____________________________________________

***Time allowed for this paper***

| Section | Reading | Working |
|---|---|---|
| ~~**Calculator-free**~~ | ~~5 minutes~~ | ~~50 minutes~~ |
| **Calculator-assumed** | 10 minutes | 100 minutes |

## Materials required/recommended for this paper

**Section Two (Calculator-assumed): 100 marks**

**To be provided by the supervisor**
Section Two Question/Answer booklet     Formula sheet

**To be provided by the candidate**
Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators satisfying the conditions set by the School Curriculum and Standards Authority for this course.

***Important Note to candidates***
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

***Instructions to candidates***

1. **All** questions should be attempted.

2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

3. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that you **do not use pencil** except in diagrams.

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**Section Two: Calculator-assumed** **(100 Marks)**

This section has **twelve (12)** questions. Answer **all** questions. Write your answers in the space provided.

Working time: 100 minutes.

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**Question 8** **(6 marks)**

A cylindrical metal disc expands when heated.

The cross-section 'height' of the disc is one fifteenth of the radius of the disc – i.e. $h = \frac{r}{15}$.

Determine the radius of the disc when the rate of changes of the volume (***V***) of the disc with respect to temperature (***t***) – i.e. $\frac{dV}{dt}$, is the same as the rate of change of the Surface Area (***SA***) or the disc with respect to temperature (***t***) – i.e. $\frac{dSA}{dt}$.

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**Question 9** **(7 marks)**

Many chemical reactions follow Wilhelmy's Law which states that the velocity of the reaction is proportional to the concentration of the reacting substance – i.e. if $a$ is the initial concentration of the reagent and $x$ is the amount transformed at time $t$ then:

$$\frac{dx}{dt} = k\left(a - x\right), \quad 0 \le x \le a$$

(a) solve this differential equation and express $x$ in terms of $kt$, for $t \ge 0$ given $a = 10$ and $x = 0$ when $t = 0$. [3]

(b) determine $k$ given $x = 4$ when $t = 2$ and hence re-write your expression for $x$ in terms of just $t$. [2]

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**Question 9** *continued*

(c) determine the amount of the substance left (un-transformed) after 5 minutes. [2]

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**Question 10** **(8 marks)**

At noon, spy drone ***A*** takes off 20 km north of Peeko town site with a velocity vector of $5\underset{\sim}{i} - 16\underset{\sim}{j} + 0 \cdot 4\underset{\sim}{k}$ km/h. Also at noon, spy drone ***B*** takes off 20 km east of Peeko town site with a velocity vector of $-16\underset{\sim}{i} + 5\underset{\sim}{j} + 0 \cdot 4\underset{\sim}{k}$ km/h. The vectors $\underset{\sim}{i}$, $\underset{\sim}{j}$ and $\underset{\sim}{k}$ are unit vectors in the directions East, North and vertically upwards respectively.

(a) Determine the position vectors of both spy drones, (***A*** and ***B***), 1 hour after take-off. [2]

(b) How far apart are the spy drones after 1 hour? [1]

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**Question 10** *continued*

(c) (i) determine the minimum distance between the two spy drones and when this occurs. [3]

(ii) interpret your result [1]

(d) In light of the result you obtained in part (c), how would you now interpret your answer in part (b)? [1]

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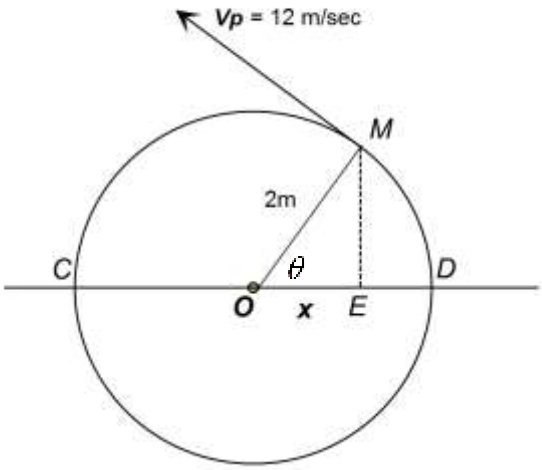
See next page

**Question 11** **(7 marks)**

A mini-motorbike is attached to a metal stake at the centre of a circular track. The mini-motorbike (***M***) starts at the point ***D*** and moves on a circular path with a constant speed of 12 m/sec. The radius of the track is 2m.




The electric power-cell (point ***E***) is attached to the 'diameter' (***CD***) – the *x*-axis. The electric power-cell travels from ***D*** to ***C*** and then from ***C*** back to ***D*** as the mini-motorbike completes one lap.

The displacement of the electric power-cell (point ***E***) from the centre stake (point ***O***) is represented by ***x***.

(a) determine:

(i) the value of $\theta$ after 1 second and hence write an equation for $\theta$ in terms of time *t*. [2]

(i) an expression for ***x*** – the displacement of ***E*** with respect to time (***t***). [1]

(ii) an expression for ***v*** – the velocity of ***E*** with respect to time (***t***). [1]

(iii) an expression for ***a*** – the acceleration of ***E*** with respect to time (***t***). [1]

(b) hence describe the motion of the point ***E***.
Use your results from above to justify your conclusion. [2]

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**Question 12** **(10 marks)**

(a) Find the square roots of $-15-8i$. [4]

(b) Suppose that $z$ and $w$ are complex numbers such that $z+w=1$ and $|z|=|w|$.

Show that $\text{Re}(z)=\text{Re}(w)=\frac{1}{2}$ and $\text{Im}(z)=-\text{Im}(w)$. [6]

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**Question 13** **(10 marks)**

The adult length of a species of fish is known to be normally distributed with a mean length of 75cm and standard deviation of 15cm. It is suspected that a random sample of 50 adult fish of mean lengths of 80cm, belong to the same species.

(a) Determine at a 1% level of significance if this suspicion is true. (3)

(b) Determine the level of the significance between the mean length of this sample and the known length. (4)

A new sample of n fish is going to be collected.

(c) Find n if the probability that the error between the sample mean and the true mean is no more than 4 cm is 0.97. (3)

D O N O T W R I T E I N T H I S A R E A

**Question 14** **(14 marks)**

The number of mobile phones, N, owned in a certain community after t years, may be modelled by $\ln(N) = 6 - 3e^{-0.4t}, \quad t \geq 0$

(a) Verify by substitution that $\ln(N) = 6 - 3e^{-0.4t}$ satisfies the differential equation

$$\frac{1}{N}\frac{dN}{dt} + 0.4\ln(N) - 2.4 = 0$$

(3)

(b) Find the initial number of phones owned in the community. (1)

(c) Find the limit $\ln(N)$ as t approaches infinity. (1)

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**Question 14** *continued*

(d) Using the mathematical model, find the limiting number of mobile phones that would eventually be owned in the community.

(2)

The differential equation in part (a) can also be written in the form $\frac{dN}{dt} = 0.4N(6 - \ln(N))$

(e) Find $\frac{d^2N}{dt^2}$ in terms of N and ln(N)

(3)

(f) The graph of N as a function of t has a point of inflection. Find the co-ordinates of the value of this point. Give the value of t correct to one decimal place and the value of N correct to the nearest integer.

(2)



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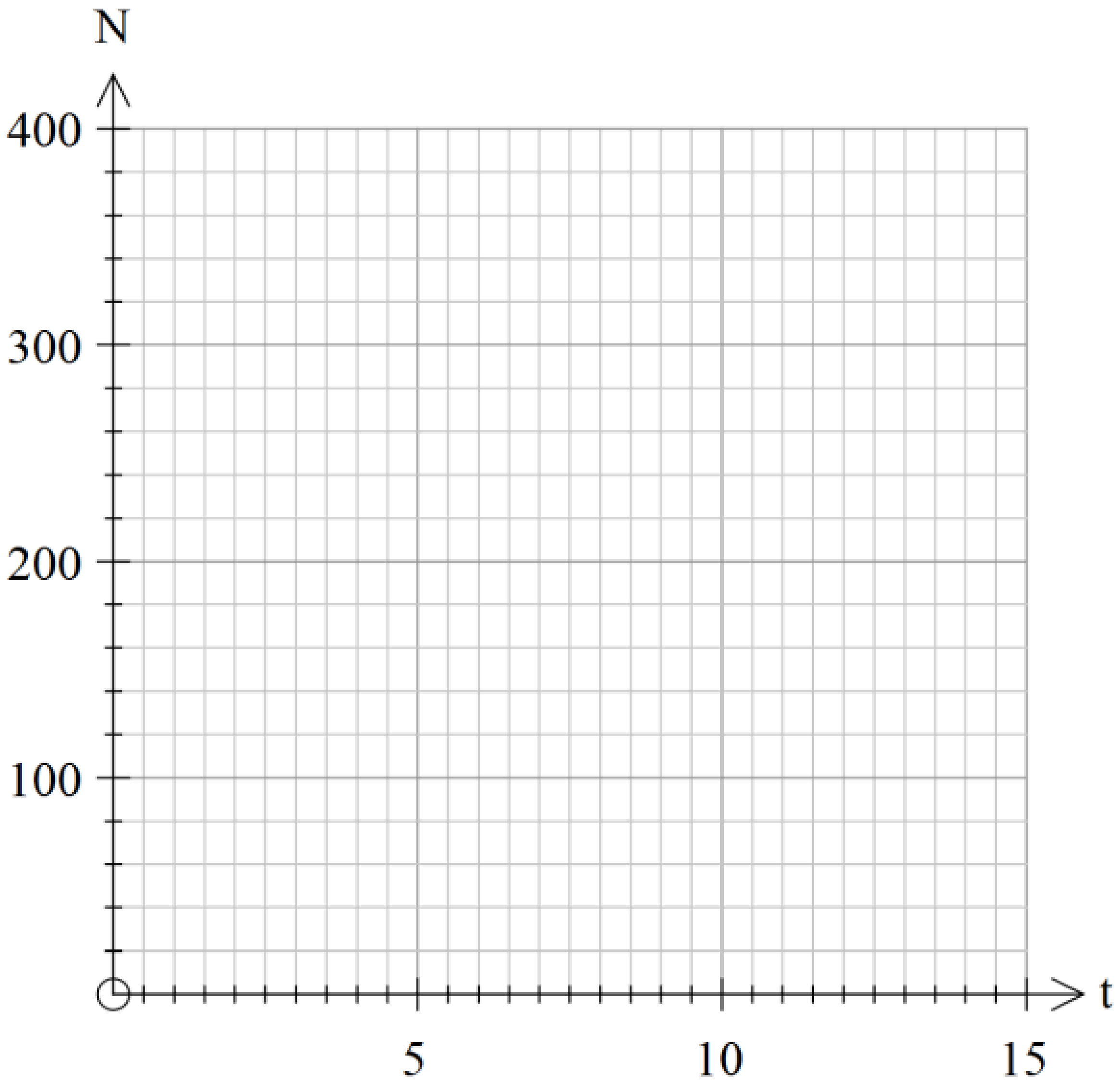
**Question 14** *continued*

(g) Sketch the graph of N as a function of t for $0 \le t \le 15$ (2)

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**Question 15** **(13 marks)**

(a) Two vehicles A and B leave a checkpoint at the same time. The first vehicle drives North at a speed of 50 km/h. The second vehicle travels at a speed of 65 km/h on a bearing of 060°.

(i) Let $\underset{\sim}{r}_A$ represent the position vector of the first vehicle at any time $t$ and let $\underset{\sim}{r}_B$ represent the position vector of the second vehicle at any time $t$.
Determine expressions for $\underset{\sim}{r}_A$ and $\underset{\sim}{r}_B$. [2]

(ii) Hence, or otherwise, find the constant rate of change of the distance between the two vehicles and interpret your result. [3]

(b) A third vehicle, travelling on a path $\underset{\sim}{r}_C = 4\cos(\pi t)i - 3\sin(\pi t)j$ leaves its starting point at the same time as the other two vehicles.

(i) Find the distance that vehicle C is from the other two vehicles at the start. [2]

**Question 15** *continued*

(ii) Find the Cartesian equation that would represent the path of vehicle C.

[2]

(iii) Find the maximum speed and the first time this occurs.

[2]

(iii) Find the distance the vehicle C travels in the first 2 seconds.

[2]

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**Question 16** **(6 marks)**

(a) When a current $I$ flows through an instrument the needle on the instrument rotates at an angle of $\theta$ $(\theta \neq 0)$ according to the function $I = \omega \tan\theta$, where $\omega$ is a constant.
A small error $\delta\theta$ is made in reading the instrument.

(i) Show that the resulting fractional error in $I$ is given by: $\dfrac{\delta I}{I} = \dfrac{2\delta\theta}{\sin 2\theta}$ [4]

(ii) For a given $\delta\theta$, find the smallest positive value of $\theta$ that minimises the fractional error $\dfrac{\delta I}{I}$. [2]

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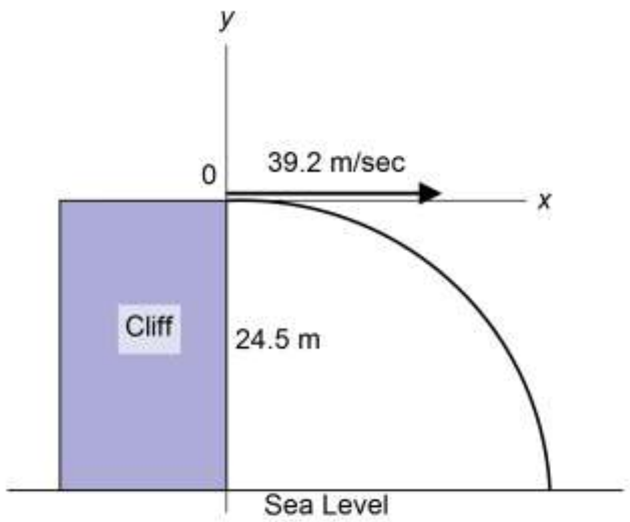
**Question 17** **(6 marks)**

(a) A stone is projected horizontally, from the top of a cliff 24·5 m high. [3]

The *"initial"* or $t = 0$ conditions, when $x = 0$; and $y = 0$; are:

$$\frac{dx}{dt} = 39 \cdot 2 \text{ m/sec}; \quad \frac{dy}{dt} = 0 \text{ m/sec};$$
$$\frac{d^2x}{dt^2} = 0 \text{ m/sec}^2; \quad \frac{d^2y}{dt^2} = -9 \cdot 8 \text{ m/sec}^2$$




Determine the parametric equations of the path of the stone after $t$ seconds.

(b) Given that $y = \frac{4}{3}\pi x^3$,find the percentage change in $y$ when $x$ increases by $0 \cdot 1\%$ – i..e. $\delta x = 0 \cdot 001x$. [3]

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**Question 18** **(7 marks)**

The points $P$, $Q$ and $R$ represent the complex numbers $z$, $2\mathrm{i}z$ and $(z+2\mathrm{i}z)$ respectively in an Argand diagram with origin $O$.

(a) Given that $z = x + iy$, where $x > 0$ and $y > 0$, sketch the Argand diagram. [3]

(b) Deduce the value of $\angle POQ$ and the geometrical relationship between the points $O$, $P$, $Q$ and $R$. [1]

**End of questions**

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**Question 18** *continued*

(c) Show that, if $y = 2x$, the point representing $z^2$ is collinear with the points *O* and *R*. [3]

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**Question 19** **(6 marks)**

**I** A sample of 30 is chosen from a binomial distribution with $p$ = 0.7 and $n$ = 20 and a graph of the sample is drawn.

**II** 20 samples of 30 are chosen from a binomial distribution with $p$ = 0.7 and $n$ = 20 and a graph of the means of each sample is drawn.

**III** 20 samples of 80 are chosen from a binomial distribution with $p$ = 0.7 and $n$ = 20 and a graph of the means of each sample is drawn.

How will the graphs from parts **I**, **II** and **III** be likely to compare. You should include the mean and standard deviation of each graph in your answers?

**End of questions**

**Additional working space**

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